Breaking Subgraph Loops for Bond Graph Model Partitioning

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ABSTRACT

This paper extends a previously-developed technique to partition and reduce models in which decoupling exists. Partitions are delimited by negligible constraint equation terms, which are identified by calculating the activity of each bond during a simulation, and comparing bond activities at each 1- and 0-junction. Locally inactive bonds at a junction are replaced with a modulated source in a process referred to as conditioning the model. If the conditioned model consists of separate bond graphs joined by modulating signals that all flow from one graph to the other, then the model can be partitioned into driving and driven subsystems.

The algorithm has been improved to handle pre-existing modulating signals that carry information from the driven to the driving dynamics, thus creating a “subgraph loop”. Subgraph loops are broken with the aid of Taylor Series expansions, constitutive law separation, and separation of signals into nominal and varying components. The original modulated bond graph elements are replicated into modulated and non-modulated elements. Elimination of the newly-created modulated elements breaks the subgraph loop and allows partitioning to proceed. The algorithm extension is applied to a medium-duty truck model to break subgraph loops from driven pitch dynamics to driving longitudinal dynamics.

1 INTRODUCTION

Model reduction is essential for efficient simulation-based design of dynamic systems. By eliminating unnecessary complexity from a model, the analyst reduces the size of the parameter estimation problem, improves computation time, and increases insight into interactions within the system (if the reduced model retains physically meaningful parameters).

In addition to eliminating unnecessary complexity from a full model, the modeler may wish to systematically determine if boundaries or partitions exist in the model that allow creation and use of two or more simpler submodels. A priori assumptions of one-way coupling within a system are often made to achieve model simplification. The intuition of the analyst may allow him or her to recognize extreme conditions that rule out a decoupled model; however, even the assumptions of experienced modelers may break down as the system parameters or environment change during the design cycle. A quantitative measure was required to monitor the validity of underlying modeling assumptions. In response to this need, the authors [1] developed a method for approaching an arbitrary lumped parameter model, systematically and quantitatively searching for local one-way coupling among individual elements, and finally determining if the local decoupling sites allow the model to be partitioned into “driving” and “driven” subsystems.
Decoupling or partitioning can lead to significant model reduction [2] and the possibility of parallel simulation of the driving and driven dynamics. Partitioned and reduced models retain a subset of the full model parameters, thus allowing physical interpretation of the resulting analysis.

The authors [3] described the application of the algorithm to a pitch plane truck model for which a predominantly longitudinal set of dynamic elements drove a partition comprised primarily of elements related to bounce and pitch (vertical and rotational) motion. Forward motion was not significantly affected by vehicle pitch for operation on a smooth road, but two-way coupling was shown to be necessary to model vehicle motion on a rough road.

The present paper addresses a limitation of the previous algorithm for the case when active bonds for modulated elements carry information from a driven dynamics to driving dynamics. Such a modulating signal creates a “subgraph loop”, and strict one-way coupling cannot be assumed. The present paper gives a procedure to systematically break subgraph loops, thereby allowing one to determine whether or not one-way power flow and information flow can be assumed in the course of model reduction.

The next section reviews the algorithms for assessing decoupling within a bond graph model and identifying driving and driven subgraphs. Section 3 describes the algorithm extension, and in Section 4 the vehicle dynamics example is revisited and the subgraph loops are broken to confirm partition viability. Discussion, summary and conclusions follow in Sections 5 and 6.

2 DECOUPLING SEARCH AND PARTITIONING – REVIEW AND LIMITATIONS

2.1 Local Activity and Bond Conditioning

The power-conserving junction structure of a bond graph model represents the constraint equations that link the constitutive law variables of energetic elements. Decoupling among elements creates negligible constraint equation terms, which manifest themselves as bonds with relatively low power flow at a 1- or 0-junction [3]. Aggregate power flow along a bond over the time interval $[t_1, t_2]$ is correlated with “activity” $A$ [4]:

$$A = \int_{t_1}^{t_2} |P| \, dt \tag{1}$$

where $P$ is instantaneous power (product of effort and flow) of the element or bond. The integration limits in Eq’n (1), which give the time window over which activity is calculated, are chosen by the analyst based on the modeling objectives. The modeler may wish to simulate the full model once, prior to calculating activities, to check for periods of transient or steady state behavior that suggest natural time windows. For example, a truck model hitting a pothole will show a pitch angle transient. A reduced model to predict the pitch response can be generated by taking a time window around the transient [5]. If the modeling goal is to predict fuel economy, then the time window can be the entire event, and the activity analysis can show whether or not the aggregate effects of surface undulations require that pitch elements be included to predict forward velocity [3]. Each bond activity at a 1- or 0-junction is divided by the maximum bond activity at the junction. Bonds (constraint terms) $i$ for which

$$\frac{A_i}{A_{\text{max}}} < \varepsilon \tag{2}$$

where $\varepsilon$ is a user-defined local activity ratio threshold, are “conditioned”, or converted to a modulated source. For a 0-junction, the modulating signal is the effort from the junction, and for a 1-junction the flow modulates the source. The half-arrow direction of the modulated source is the “conditioned”, or converted to a modulated source. For a 0-junction nor the effort input to a 0-junction. Causality impedance. Note that the locally inactive bonds must be with all locally negligible power bonds converted to

Table 1. Interpretation of locally inactive bonds.

<table>
<thead>
<tr>
<th>Scenario (i)</th>
<th>Case A. $A_i &lt; A_{i\text{out}}$</th>
<th>Case B. $A_i &lt; A_{i\text{in}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{m+1}$ $e_2$</td>
<td>$M_{Se}$ $f_1$</td>
<td>$M_{Se}$ $f_1$</td>
</tr>
<tr>
<td>$f_{n1a}$ $e_2$</td>
<td>$M_{Se}$ $f_1$</td>
<td>$M_{Se}$ $f_1$</td>
</tr>
<tr>
<td>$f_{n1b}$ $e_2$</td>
<td>$M_{Se}$ $f_1$</td>
<td>$M_{Se}$ $f_1$</td>
</tr>
<tr>
<td>$f_{n2}$ $e_2$</td>
<td>$M_{Se}$ $f_1$</td>
<td>$M_{Se}$ $f_1$</td>
</tr>
</tbody>
</table>

External bond graph elements ($I$, $C$, $R$) are represented by the symbol $Z$ in Table 1, signifying a generalized impedance. Note that the locally inactive bonds must be causally weak, i.e., they must not provide the flow input to a 1-junction nor the effort input to a 0-junction. Causality reassignment may be required.

A conditioned model or bond graph is a bond graph with all locally negligible power bonds converted to modulated sources.
2.2 Subgraphs and Partitions
Bond graph conditioning is followed by identification of subgraphs and partitions. Given a conditioned bond graph, a subgraph S is a set of elements that has no power bonds connected to any bond graph element outside the set. The subgraph may be connected to the rest of the bond graph by modulating signals, or may be unconnected. When all new modulating signals (due to bond conditioning) between two subgraphs are directed from one subgraph to another subgraph, the subgraph from which the signals originate is a driving subgraph S⁺, and the other is a driven subgraph S⁻. A driving partition is a subgraph P⁺ that is connected to at least one other subgraph strictly by modulating signals directed outwards from P⁺ to that other subgraph. A driven partition is a subgraph P⁻ that is connected to at least one other subgraph strictly by modulating signals directed inwards towards P⁻ from that other subgraph. The activity comparison and interpretation are incorporated into the algorithms presented in [3].

2.3 Implications for Model Reduction
Finding and separating partitions allows immediate model reduction even if the partitions are not subjected to techniques to eliminate non-contributing states. An output associated with a driving partition element can be predicted by simulating the driving partition by itself. To predict a driven partition output, the driving submodel can be used to generate the necessary modulating signals to excite the driven dynamics. As an alternative to sequential simulation of driving and driven partitions, the driven and driving partition outputs can be generated nearly simultaneously on different processors with information flowing from driving to driven as necessary.

2.4 Subgraph Loops
A subgraph loop SL is a set of subgraphs in which each S ∈ SL is connected to the previous by modulating signals directed into S, and to the next by modulating signals directed out of S. A subgraph loop can itself be a subgraph or partition. A driven partition must not be an element of a subgraph loop.

The distinction between subgraphs and partitions is drawn because modulating signals may exist, prior to conditioning, that create subgraph loops and prevent partitioning. A conditioned model with driving and driven subgraphs suggests that one-way power flow exists between the graphs. A subgraph loop in such a model indicates that one-way information flow from one subgraph to another cannot be assumed. For complete decoupling, driving and driven partitions must exist. A systematic approach to elimination of subgraph loops is the contribution of this paper.

3 BREAKING SUBGRAPH LOOPS: MODULATING SIGNAL ASSESSMENT
The contribution of an active bond can be evaluated if the associated modulated element can be replicated into modulated and non-modulated elements, the total effect of which is the same as the original element. If the power bonds to the modulated elements can be conditioned, then the subgraph loop is broken at that point. Three possible approaches to element replication are:
- Taylor expansion of element constitutive laws
- Analytical separation of constitutive laws
- Separation of the signal into nominal and varying components

3.1 Taylor Series Expansion
For a system that operates near an equilibrium point, the user may attempt to expand the constitutive law containing the modulating signal as a Taylor series. A separate element will result for each term of the series. The element corresponding to the first (equilibrium) term of the expansion will be non-modulated. If local comparison of the activities of the individual elements suggests elimination of all the others, then no element modulation is necessary.

Figure 1 shows a modulated transformer example in which the modulus \( \cos \theta \) is expanded about a constant \( \theta_0 \), with the linear term in \( \delta \theta \) retained. If both bonds of the MTF in the middle bond graph are locally inactive, then the MTF is eliminable and a constant-modulus TF is an acceptable approximation of the transformation. The modulating signal is thus removed.

3.2 Constitutive Law Separation
In Figure 2a, a C and I element are connected to a 1-junction. The modulated resistor constitutive law consists of clearly separable modulated and non-modulated components. The element can be broken into separate elements with the same causality as the original, affixed to a common junction – in this case the original 1-junction whose flow serves as the input to both. If as in Figure 2b the element is attached to a 0-junction, the separate \( R_0 \) and \( \Phi(x) \) elements still require a common 1-junction at which the two resistance components can be added.

3.3 Signal Separation
If the constitutive law is not separable but a nominal value of the modulating signal can be identified, then the element can be duplicated. One copy of the element incorporates the constant nominal signal into its constitutive law, while the second copy is modulated by the difference between the total and nominal signal components. A local activity comparison then determines whether the modulated copy of the element can be neglected. Note that prior to element duplication, the modulating signal variable \( x \) should be replaced with \( (x_o + \delta x) \) – the sum of the nominal value and varying component – and the constitutive law then expanded. Additional junction structure may be required to ensure that the original system equations are unchanged. Junction structure equivalences are described in [6]. Figure 3 shows examples. As before, breaking the subgraph loop at that point requires that the new MR elements be locally inactive.

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When the preceding approaches are not applicable, the user may test a simplifying assumption that breaks the subgraph loop as follows:

- Record the modulating signal from the non-partitionable model into an array $R_2$.
- Proceed with a modeling assumption that breaks the loop, repeat the simulation, and record the modulating signal into an array $R_1$.
- Create a third array $R_2-R_1$, noting that $R_1 + (R_2-R_1) = R_2$.
- Duplicate the modulated element. Modulate one copy with the pre-recorded $R_1$ array, and the other with the $R_2-R_1$ array.
- Re-run the simulation to test the local activity of the element modulated by $(R_2-R_1)$. Inactivity indicates that the subgraph loop may be broken at that site.

Figure 4a shows the conditioning algorithm of [1], and Figure 4b shows the partitioning algorithm, extended to incorporate the treatment of subgraph loops.
4 ILLUSTRATIVE EXAMPLE

4.1 Vehicle Model

The nonlinear pitch plane model of a Class VI delivery truck from [3,7] is revisited to break subgraph loops using the procedures of Section 3. Outputs of interest are forward speed \( V \) and pitch angle \( \theta \). The vehicle accelerates at full-throttle from a standstill, on a flat road on which a 1:10 slope is encountered after 1200 feet of travel. The maneuver lasts 120 seconds. The conditioned model reveals driving and driven subgraphs containing predominantly longitudinal and pitch dynamic elements respectively. Figure 5 shows a pitch plane vehicle schematic, and Figure 6 shows the conditioned bond graph generated using the 20SIM software package [8]. The subscripts \( s \) and \( t \) indicate suspension and tire, respectively, with \( f \) and \( r \) denoting front and rear. Left superscripts correspond to reference frames in Figure 5. Frame 1 is attached to the sprung mass. Frames 2 and 3 are affixed to the front and rear wheel hubs, with their \( x \) axes aligned with the road surface at each wheel. The sprung mass has mass \( M \) and inertia \( J \).

4.2 Bond Graph Conditioning Results and Subgraph Loops

The conditioned bond graph of Figure 6 is based on a 4% relative activity threshold, with activities calculated over the entire 120 second time interval. Heavy dashed lines indicate modulating signals due to bond conditioning, and dotted lines are active bonds from the original bond graph. Driving subgraph \( S' \) contains longitudinal elements such as sprung mass in the \( x \)-direction, aerodynamic drag, drive torque, tire slip and rolling resistance. Driven subgraph \( S^- \) contains the remaining elements, including all elements pertaining to sprung mass rotation and suspension deflection.

Subgraph loops arise due to the modulating signals \( F_{S'} \) (rear tire normal force) and pitch angle \( \theta \). Pitch angle

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**Figure 4a.** Conditioning algorithm.

**Figure 4b.** Partitioning w/ subgraph loops.
is the output of the pitch velocity 1-junction, and modulates the longitudinal gravity source and a coordinate transformation from frame 3 to frame 1. Rear tire normal force modulates the rolling resistance and longitudinal traction force \( R \) elements. The traction force is a nonlinear function of wheel slip and normal load.

Wheel slip arises due to tire compliance and the resulting difference between the actual forward velocity \( v \) of a wheel hub and the product of tire radius \( r \) and wheel angular velocity \( \omega \). The slip ratio \( \kappa \) and resistance forces are defined as:

\[
F_{slip} = \frac{sgn(\kappa)F_z}{\mu \kappa} \\
\kappa = \frac{r \omega - v}{v}
\]

where \( F_z = \text{tire force normal to the road}, \mu = \text{coefficient of friction}, \) and \( \kappa_{\text{max}} = \text{slip ratio at tire saturation.} \)

Rolling resistance is also a function of normal load and longitudinal velocity, along with tire inflation pressure and empirical constants \( c_r \).

4.3 Assessment of Pre-Existing Modulating Signals

As previously stated, the driving subgraph rear tire rolling and slip resistances are functions of the normal force \( F_{zr} \) from the driven subgraph. Figure 7 shows the normal force signal generated during simulation of the full model. A nominal signal value of -39950N was identified as the static tire load, and the difference between the total and nominal values gave the varying component. The method of Section 3.3 is applied.

The rolling resistance is decomposed into three elements with common input velocity, the effort outputs of which add to give the correct total resistance force. The slip resistance is decomposed into two elements joined to a 1-junction representing the slip velocity. The details of the decomposition follow.

\[
F_{\text{rolling}} = sgn(v) \left[ c_1 + c_r F_{zr} + c_s F_{\text{slip}}^2 + c_s F_{\text{slip}}^3 \right]
\]
Rolling resistance

The normal force is split into constant (nominal) and varying components $F_{z1}$ and $F_{z2}$ such that

$$F_{z1} = -39950$$
$$F_{z2} = F_z + 39950$$  (6)

When $F_z$ is replaced with $F_{z1}+F_{z2}$ in the constitutive law of Eq. (5), three elements result, with the following constitutive laws:

$$R_{roll1} : F_{roll1} = \text{sgn}(v)\left[c_z F_{z1} + c_z F_{z2}^2\right]$$
$$R_{roll2} : F_{roll2} = \text{sgn}(v)\left[c_z F_{z2} + c_z F_{z2}^2\right]$$
$$R_{roll3} : F_{roll3} = \text{sgn}(v)\left[2c_z F_{z2} F_{z2}^2\right]$$  (7)

$R_{roll1}$ is a non-modulated element in the sense that the varying component of $F_z$ from the driven partition is not required. If a local activity comparison shows $R_{roll2}$ and $R_{roll3}$ inactive, then the elimination of the modulating signal is justified. Had one of the terms in the first of Eq’ns (7) contained $F_{z2}$, then the technique of Section 3.2 could be applied to separate the constitutive law.

Slip resistance

Modulated and non-modulated elements are created with the following constitutive laws:

$$R_{slip1} : F_{slip1} = \frac{\text{sgn}(\kappa)F_z |\mu| \kappa}{\kappa_{\text{max}}}$$
$$R_{slip2} : F_{slip2} = \frac{\text{sgn}(\kappa)(- F_z \mu \kappa)}{\kappa_{\text{max}}}$$  (8)

$R_{slip1}$ uses the constant nominal normal force to calculate $F_{slip}$. $R_{slip2}$ is a function of the varying portion of $F_z$, with the absolute value of $F_z$ replaced with $-F_z$ to reflect the fact that normal force magnitudes less than 39950 create lower tractive forces. The modulated element force must thus be subtracted from the force of $R_{slip1}$ at the common 1-junction.

The replicated elements and relative activities of the bonds are shown below in Figure 8. The modulated rolling and slip resistance elements may be eliminated based on the 4% threshold.

Pitch angle

The pitch angle $\theta$ from the driven subgraph modulates both the transformer $\cos(\theta-\theta_i)$ and the gravity source $-Mgsin\theta$ in Figure 6. Loop breakage is attempted by assuming that the DC component of pitch angle follows the road inclination, except for a small offset $\theta_i$ equal to the vehicle’s pitch attitude while at rest. For the Class VI truck $\theta_i = -0.007$ rad. The nominal angle $\theta-(\theta_i+\theta)$ is given a zero value in the following expansion:

$$\cos(\theta - \theta_i) \approx \cos(\theta - \theta_i - \theta_i) \approx 1 - \frac{1}{2}(\theta - \theta_i - \theta_i)^2$$  (9)

The transformer $\cos(\theta-\theta_i)$ is replaced by a transformer with unity modulus and a second transformer modulated by $\theta$. Figure 9 shows that both bonds of the $MTF$ are locally inactive, allowing elimination of the modulated element.
4.4 Model Output and Computation Time

Computation time savings due to partitioning and breaking of algebraic loops through conditioning, are reported in [2]. For the vehicle model herein, simulating the driving and driven partitions separately and sequentially to generate the outputs of interest takes approximately the same time as simulating the conditioned model. However, partitioning does allow simulation of only one partition at a time on a given processor, or parallel processing of completely decoupled submodels. For larger models such as the engine model of [2], partitioning can cut simulation time significantly compared to conditioning.

Figure 11 compares velocity and pitch angle predictions for the conditioned and partitioned models. The similarity of the results validates breaking the subgraph loop through element replication and local activity calculation.

5 DISCUSSION

While initial overhead is required to construct a high-fidelity ‘full’ model for initial activity calculation, the effort is justified for design processes in which an optimization algorithm may execute a reduced or partitioned model thousands of times. With the proposed algorithm extension, the full system must be simulated once to determine activities and condition the model, and then a second time if subgraph loops are present and elements are to be replicated and re-tested for inactivity. Given that research into determining the range of validity of parameters and inputs over which partitioned models are valid is ongoing, the full model may have to be revisited periodically. Aggregate computation time will still be significant even if an occasional run must be made with a model that is needlessly complex, but is used to check or tune the partitioned model.

The physical insight gained (or retained) by the partitioning algorithm is a valuable attribute of the technique. While having the advantage of applicability to nonlinear systems, energy-based methods in the proper modelling field are not easily subject to rigorous mathematical proof. The nonlinear nature of the activity metric, for example, inhibits closed-form relations between local activity ratio and change in performance of a model as bonds are conditioned. Theoretical justification of the energy-based proper modeling area is a work in progress [9].

6 SUMMARY AND CONCLUSIONS

This paper describes the extension and refinement of a procedure for identifying decoupling within, and reducing, bond graph models by searching for negligible constraint terms based on low relative activity. Negligible constraint terms are removed by “conditioning” the model, after which subgraphs and partitions are identified. The present paper improves the partition identification stage by more formally treating the removal of “subgraph loops” – conditioned bond graphs exhibiting one-way power flow between driving and driven dynamics, but two-way information flow due to pre-existing modulated elements such as coordinate transformers and modulated resistors. Through series expansion of constitutive laws, separating modulating signals into nominal and varying components, and use of bond graph equivalences, modulated elements are replicated into modulated and non-modulated elements. If local inactivity suggests removal of all new modulated elements, then the loop is broken.

Subgraph loops were systematically broken for a conditioned Class VI truck model in which driving longitudinal dynamics were modulated by the pitch angle and rear tire normal force from the driven dynamics. The ongoing evolution of the method increases the ease with which decoupling can be found, and models reduced, by an engineer without vast domain expertise or intuition.
Even a comparative expert, who may not realize when an initially conservative decoupling assumption about a complex system breaks down, can benefit from the conditioning and partitioning algorithms.

7 REFERENCES


[8] 20SIM (2003), v.3.6, Controllab Products b.v., Enschede, Netherlands.


BIOGRAPHIES

Geoff Rideout is an Assistant Professor at Memorial University in St. John’s, Newfoundland, Canada. He received his Bachelor’s degree in mechanical engineering there in 1993, and worked in industry for three years before earning a Master of Science in engineering at Queen’s University at Kingston in 1998, and his Ph.D. at the University of Michigan in 2004. He has worked as a lecturer and Post-Doctoral Fellow in the Automated Modeling Laboratory at the University of Michigan, and as a lecturer at the Humber Institute of Applied Technology in Toronto. He is on the International Program Committee of the ICBGM. His research interests are bond graphs, proper modeling, simulation-based design of dynamic systems, vehicle dynamics, and multibody dynamics.

Jeffrey L. Stein is a Professor of Mechanical Engineering at the University of Michigan, and director of the Automated Modeling Laboratory. His specialty is in the use of computer based modeling and simulation tools for system design and control. He has particular interest in Proper Models, or more specifically algorithms for automating the development of dynamic mathematical models with physical state variables and parameters that have the minimum yet sufficient complexity. His application areas include: vehicle dynamics, vehicle mobility, and machine tool spindles. He has authored or co-authored over 100 articles in journals and conference proceedings. He has been on faculty at the University of Michigan since 1983, and became a Full Professor in 1996. He is an Associate Director of the Automotive Research Center (ARC), and a former chair of the Executive Committee of the Dynamic Systems and Control Division of ASME.