A SIMPLE TWO-PHASE FRICTIONAL MULTIPLIER CALCULATION METHOD

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ABSTRACT

In this paper, a simple method for calculating two-phase frictional multiplier for total flow assumed liquid in the pipe ($\phi_0^2$) is presented. The homogeneous model is used to calculate the fluid properties (density and viscosity). The Churchill model is used to define the Fanning friction factor to take into account the effect of the mass flux on $\phi_0^2$. Effect of stream pressure on $\phi_0^2$ is also investigated. It is found that $\phi_0^2$ decreases with increasing the stream pressure at a given mass quality and reaches 1 at the critical pressure. On the other hand, it is found that $\phi_0^2$ increases with increasing the mass flux at a given mass quality. Comparison with other existing correlations for calculating $\phi_0^2$ such as the Wallis correlation based on the homogeneous model without mass effect on $\phi_0^2$, the Martinelli-Nelson correlation, the Chisholm correlation, and the Friedel correlation is presented. When the mass flux value becomes low, the effect of mass flux on $\phi_0^2$ becomes small and present correlation approaches the Wallis correlation. Both the present correlation and the Wallis correlation approach the maximum two-phase frictional multiplier in a smooth consistent manner while the other correlations show a peaking effect at high mass qualities. The Friedel correlation shows better agreement with the present correlation than both the Martinelli-Nelson correlation and the Chisholm correlation. Comparison with other experimental data shows better agreement with the present correlation than the Martinelli-Nelson correlation.

INTRODUCTION

The pressure drop in two-phase gas-liquid flow is an important design parameter in many engineering applications. Due to its importance, numerous investigations on this topic can be found in the literature. Examples of engineering applications include: chemical industry, nuclear industry, petroleum industry, refrigeration and air-conditioning applications, and space station applications.

Total pressure drop for two-phase flow in pipes consists of frictional, acceleration, and gravitational components. It is necessary to know the void fraction (the ratio of gas flow area to total flow area) to compute the acceleration, and gravitational components. To compute the frictional component of pressure drop, either the two-phase friction factor or the two-phase frictional multiplier must be known [1].

The research on pressure drop in two-phase flow began in the 1940's. Since then, pressure drop and holdup data have been collected for horizontal, vertical, and inclined gas-liquid systems. From the pressure drop and holdup data, many attempts have been made to develop general procedures for predicting these quantities.

There are two principal types of frictional pressure drop models in two phase flow: the homogeneous model and the separated model. In the first, both liquid and vapor phases move at the same velocity (slip ratio = 1). Consequently, the homogeneous model has also been called the zero slip model. The homogeneous model considers the two-phase flow as a single-phase flow having average fluid properties depending on mass quality. Thus, the frictional pressure drop is calculated by assuming a constant friction coefficient between the inlet and outlet sections of the pipe. In the second, two-phase flow is considered to be divided into liquid and vapor streams. Hence, the separated model has been referred to as the slip flow model. The separated model was originated from the classical work of Lockhart and Martinelli [2] that was followed by Martinelli and Nelson [3]. The Lockhart-Martinelli method is one of the simplest procedures for calculating two-phase frictional pressure drop and hold up. One of the biggest advantages of the Lockhart-Martinelli method is that it can be used for all flow patterns. However, relatively low accuracy must be accepted for this flexibility. The separated model is popular in the power plant industry. Also, the separated model is relevant for the

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prediction of pressure drop in heat pump systems and evaporators in refrigeration. The success of the separated model is due to the basic assumptions in the model which are closely met by the flow patterns observed in the major portion of the evaporators.

The basic procedure used in predicting the frictional pressure drop in two-phase flow is developing a general correlation based on statistical evaluation of the data. The main disadvantage of this procedure is the difficulty in deciding on a method of properly weighing the fit in each flow pattern. For example, it is difficult to decide whether a correlation giving a poor fit with stratified flow and a good fit with annular flow is a better correlation than one giving a fair fit for both kinds of flow. Although the researchers deal with two-phase flow problems still continue to use general correlations, alternate procedures must be developed to improve the ability to predict the frictional pressure drop. In addition, correlations fitted to data banks that contain measurements with a number of liquid-gas combinations for different flow conditions and pipe diameters often have the disadvantage of containing a large number of constants and of being inconvenient in use. The correlation developed by Bandel [4] is an example of this type of correlations.

It is clear that in spite of the considerable progress made to predict the frictional pressure drop experimentally and theoretically [5-7], a need still exists for accurate, convenient, and rapid estimation procedures.

The two-phase frictional pressure drop ($\Delta p_f$) can be expressed in terms of the single-phase frictional pressure drop for the total flow considered as liquid ($\Delta p_{f,lo}$) using two-phase frictional multiplier for total flow assumed liquid in the pipe ($\phi_{lo}^2$). This representation method is often useful for calculation and comparison needs. The single-phase frictional pressure drop for the total flow considered as liquid is computed from the total mass flux ($G$) and the physical properties of the liquid. The concept of all-liquid frictional pressure drop is useful because it allows the correlation to be tied into single-phase results at one end and eliminates any ambiguity about the physical properties to use, especially viscosity. Moreover, the all-liquid frictional pressure drop is chosen over the all-gas frictional pressure drop, because the liquid density generally does not vary in a problem, while the gas density changes with pressure. Also, the correlation of frictional pressure drop in terms of the parameter ($\phi_{lo}^2$) is more convenient for boiling and condensation problems than ($\phi^2$). The parameter ($\phi_{lo}^2$) was first introduced by Martinelli and Nelson [3] in 1948.

In the present study, the homogeneous model is used to calculate the fluid properties (density and viscosity). The Churchill model is used to define the Fanning friction factor. This study is undertaken to investigate the effects of stream pressure and mass flux on $\phi_{lo}^2$.

**LITERATURE REVIEW**

Martinelli and Nelson [3] presented a tentative method for the calculation of the pressure drop during forced circulation boiling of water. Their method was based upon the application of pressure drop data, obtained during the isothermal flow of air and different liquids, to the calculation of local pressure gradients during forced circulation boiling. They assumed that the flow regime would always be 'turbulent-turbulent' since any normal forced circulation boiler design for all practical purposes would involve this flow mechanism only. Also, they assumed that the static pressure drop of the liquid phase was the same as that of the vapor phase. Because of the nature of this assumption, their model would be better suited to annular flow. During the isothermal flow in a horizontal pipe, the frictional pressure drop only was obtained, since no change in the acceleration pressure drop took place. Therefore, the extension of the isothermal data to the case of forced circulation boiling gave the frictional pressure drop. They developed a correlation for calculating $\phi_{lo}^2$. They defined a two-phase frictional multiplier for total flow, which assumed liquid only in the pipe ($\phi_{lo}^2$) as follows:

$$\phi_{lo}^2 = \frac{(dp / dz)_f}{(dp / dz)_{f,lo}}$$

The Martinelli-Nelson correlation was empirical. They presented their correlation in a graphic manner. On the horizontal axis, the independent parameter was the mass quality ($x$). On the vertical axis, the dependent parameter was the ratio of two-phase frictional pressure drop to all-liquid frictional pressure drop ($\phi_{lo}^2$). When the flow was all liquid, $x$ was equal to 0, and $\phi_{lo}^2$ was equal to 1. When the flow was all gas, $x$ was equal to 1, and $\phi_{lo}^2$ was equal to $\Delta p_{f,lo}/\Delta p_{f,g}$, so the pressure drop was equal to $\Delta p_{f,g}$. On the grid, they plotted a family of curves for pressures from 100 psia (6.89 bar) to 3 206 psia (221.2 bar) (the critical pressure). They found that $\phi_{lo}^2$ decreased by increasing pressure at a given $x$, and reached 1 at the critical pressure. From these curves, the frictional pressure drop during forced circulation boiling could be estimated quickly once the exit mass quality, the boiling pressure, and the pressure drop for 100% liquid were known. They compared the predicted pressure drop with the measured pressure drop for the pressure range from 18 to 3 000 psia and for the exit mass quality range from 4 to 100%. The comparison indicated that the method had promise. However, this method was based upon a meager amount of data. So, further experimental verification before this method could be considered valid.

Lockhart and Martinelli [2] presented data for the simultaneous flow of air and liquids including benzene, kerosene, water, and different types of oils in pipes varying in diameter from 0.0586 in. to 1.017 in. They developed their method for pressures from atmospheric to 50 psi. There were four types of isothermal two-phase, two-component flow. In the first type, flow of both the liquid and the gas were turbulent. In the second type, flow of the liquid was laminar and flow of the gas was turbulent. In the third type, flow of the liquid was turbulent and flow of the gas was laminar. In the fourth type, flow of both the liquid and the gas were laminar. They correlated the pressure drop resulting from these different flow mechanisms by means of the Lockhart-Martinelli parameter ($X$). The Lockhart-Martinelli parameter ($X$) was defined as:

$$X^2 = \frac{(dp / dz)_{f,g}}{(dp / dz)_{f,lo}}$$

In addition, they expressed the two-phase frictional pressure gradient in terms of factors, which multiplied single-phase
The deviation for the comparison was 10%.

with the later data of Haywood et al. [11] for a vertical tube heat balance. He based these curves directly on the experimental results of the boiler circulation research sponsored at the University of Cambridge by the Water-Tube Boilers’ Association. He compared his calculated values with the later data of Haywood et al. [11] for a vertical tube. The deviation for the comparison was 10%.

They presented the relationship of \( \phi_i \) and \( \phi_g \) to \( X \) in graphical forms. They proposed tentative criteria for the transition of the flow from one type to another. They correlated the percent of pipe filled with liquid under any flow conditions for all four-flow types by means of the Lockhart-Martinelli parameter \( X \). Although the Lockhart-Martinelli correlation related to the adiabatic flow of low pressure air-liquid mixtures, they purposely presented the information in a generalized form to enable the application of the model to single component systems, and, in particular, to steam-water mixtures. Their empirical correlations were shown to be as reliable as any annular flow pressure drop correlation [8]. The disadvantage of this method was its limit to small-diameter pipes and low pressures because many applications of two-phase flow fell beyond these limits.

Isbin et al. [9] studied frictional pressure drop of steam-water mixtures for adiabatic flow in horizontal pipes of 0.484 and 1.062 in. diameter respectively. The range of pressure was from 25 to 1 415 psia. They carried out the low pressure experiments (25 to 100 psia) in 1.062 in. diameter. The range of the intermediate pressure was from 400 to 800 psi. They carried out the high pressure experiments (1 000 to 1 415 psia) in 0.484 in. diameter. The total mass flow rate range was from 454 to 4 350 lb/hr. In their experiments, they used 3 ranges of inlet steam flow rates; low (330 to 434 lb/hr), medium (800 to 900), and high (1 250 to 1 350). They varied the mass quality from about 8 to 98%. They calculated the mass quality in the test section from an energy balance. They synthesized the steam-water mixtures by mixing steam and water. They took considerable care to ensure that the method of mixing did not affect the pressure drop results and that the pressure measurements were made far from the inlet and outlet sections of the pipe. They compared the data to standard correlations. They suggested a new restricted correlation that took into account the pressure and flow rate dependencies.

Thom [10] gave a simplified scheme for the calculation of pressure drop during forced circulation of a two-phase mixture of boiling water and steam. His method followed that proposed by Martinelli and Nelson [3], which had been extended to include the gravitational term in vertical evaporating tubes. He assumed that water entering the tube was at saturation temperature. Thus evaporation, with net generation of steam, started at once. He gave curves from which frictional, acceleration, and gravitational losses could be estimated provided the outlet mass quality had been calculated from a heat balance. He based these curves directly on the experimental results of the boiler circulation research sponsored at the University of Cambridge by the Water-Tube Boilers’ Association. He compared his calculated values with the later data of Haywood et al. [11] for a vertical tube. The deviation for the comparison was 10%.

Baroczy [12] described a systematic correlation for the prediction of two-phase friction pressure drop for both single component flow and two-component flow. The correlation considered fluid properties, mixture mass quality, and mass flux. The correlation was based on data for steam, water-air, and mercury-nitrogen for a wide range of mass quality, and mass flux. He called liquid to gas viscosity and density ratio \((\mu_l/\mu_g)^{0.2}/(\rho_l/\rho_g))\) as the property index. The property index had the advantage of not requiring knowledge of the critical pressure and temperature in order to establish the property ratios at the critical point, where they had a value of 1. By similar reasoning, it could be used to establish the analogous condition for two-phase, two-component flow, that was, equal viscosity and density for each phase. Thus, the physical properties of single and two-component, two-phase fluids could be described on the common basis. The two-phase frictional multiplier for total flow assumed liquid in the pipe \((\phi_{t,2}^2)\) was shown to be a function of property index, mixture mass quality, and mass flux. He varied the property index from critical pressure to five decades below. He varied the mixture mass quality from 0.1 to 100%. He varied the mass flux from 0.25x10^6 to 3x10^9 lb/hr. He showed that the reciprocal of the property index \(\phi_{t,1} = (dp/dz)_{t,1}/(dp/dz)_{t,0,1}\) was a constant (i.e. \(\phi_{t,2}^2/\phi_{t,1}^2\)). He presented some calculated values for steam-water flows in a table. His table showed that \(\phi_{t,1}^2\) decreased with increasing pressure at a given \(x\) and had a value of 1 at the critical point. He found that his method worked quite well for dispersed phase flows (bubbly flows) but tended to underpredict the pressure drop for separated flows. The Martinelli-Nelson correlation tended to be better for separated flows. This was perhaps to be expected since the Wallis correlation based on the homogeneous model assumed a fully dispersed flow while the Martinelli-Nelson correlation was based on a separated (annular) flow concept.

Chisholm [13] developed equations in terms of the Lockhart-Martinelli correlating groups for the friction pressure gradient during the flow of gas-liquid or vapor-liquid mixtures in pipes. His theoretical development was different from previous treatments in the method of allowing for the interfacial shear force between the phases. Also, he avoided some of the
anomalies occurring in previous “lumped flow”. He gave simplified equations for use in engineering design. His equations were:

\[ \phi^2 = 1 + \frac{C}{X} + \frac{1}{X^2} \]  

(6)

\[ \phi^2 = 1 + CX + X^2 \]  

(7)

The values of \( C \) were dependent on whether the liquid and gas phases were laminar or turbulent flow. The values of \( C \) were restricted to mixtures with gas-liquid density ratios corresponding to air-water mixtures at atmospheric pressure. The different values of \( C \) were given in the Table 1.

**Table 1. Values of \( C \) for Different Flow Types**

<table>
<thead>
<tr>
<th>Liquid</th>
<th>Gas</th>
<th>( C )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Turbulent</td>
<td>Turbulent</td>
<td>20</td>
</tr>
<tr>
<td>Laminar</td>
<td>Turbulent</td>
<td>12</td>
</tr>
<tr>
<td>Turbulent</td>
<td>Laminar</td>
<td>10</td>
</tr>
<tr>
<td>Laminar</td>
<td>Laminar</td>
<td>5</td>
</tr>
</tbody>
</table>

He compared his predicted values using these values of \( C \) and his equation with the Lockhart-Martinelli values. He obtained good agreement with the Lockhart-Martinelli empirical curves.

Chisholm [14] studied the influence of mass flux on friction pressure gradients during the flow of steam-water mixtures in rough and smooth tubes. He obtained the data at pressures between 30 bar (435 psia) and 175 bar (2 540 psia). He obtained the data at mass fluxes between 280 and 20 000 kg/m².s. He obtained the experimental data at mass fluxes below 800 kg/m².s with 48 mm bore tubes. He obtained the experimental data at mass fluxes between 800 and 2 000 kg/m².s with 8 mm tubes. He obtained the experimental data at mass fluxes above 2 000 kg/m².s with 1-2.6 mm tubes. Data obtained at 800 kg/m².s with both the 48 mm and 8 mm tubes did not indicate a significant effect of diameter. From analysis of data, he developed equations for friction pressure gradient. His equations allowed for the influence of the ‘mass flux effect’, not previously allowed for in accepted correlations. He gave simplified equations for use in engineering design. His equations:

\[ \phi^2 = 1 + (\Gamma^2 - 1)(Bx^{2-n/2}(1-x)^{2-n/2} + x^{2-n}) \]  

(10)

Also, the above equation could be used to transform the graphical procedure of Baroczy. The values of \( B \) corresponding to Baroczy correlation were approximated by the following equations:

\[ B = \frac{55}{G^{1/2}} 9.5 \leq \Gamma < 9.5 \]  

(11)

\[ B = \frac{520}{IG^{1/2}} 9.5 \leq \Gamma < 28 \]  

(12)

\[ B = \frac{15000}{I^2G^{1/2}} \Gamma \geq 28 \]  

(13)

There was evidence that the Baroczy correlation might underestimate the prediction of friction in certain situations, and for this reason, the values of \( B \) in Table 2 were recommended (also depending on the mass flux \( G \)).

**Table 2. Values of Parameter \( B \) for Smooth Tubes**

<table>
<thead>
<tr>
<th>( \Gamma )</th>
<th>( G ) (kg/m².s)</th>
<th>( B )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 &lt; ( \Gamma ) ≤ 9.5</td>
<td>( G \leq 500 )</td>
<td>4.8</td>
</tr>
<tr>
<td>9.5 &lt; ( \Gamma ) &lt; 28</td>
<td>( 500 &lt; G &lt; 1900 )</td>
<td>520/( IG^{1/2} )</td>
</tr>
<tr>
<td>( \Gamma \geq 28 )</td>
<td>( G \geq 600 )</td>
<td>15000/( I^2G^{1/2} )</td>
</tr>
</tbody>
</table>

Chisholm [16] showed that his previous equation for predicting the friction pressure gradient during two-phase flow, Eq. (6), was an unsatisfactory form for use with evaporating flows as \( \frac{dp}{dz} \), in that case, varied along the flow path. That equation could be transformed with sufficient accuracy for engineering purposes to

\[ \phi^2 = 1 + (\Gamma^2 - 1)\left[ Bx^{2-n/2}(1-x)^{2-n/2} + x^{2-n} \right] \]  

Chisholm [15] defined a physical property coefficient \( \Gamma \):

\[ \Gamma = \left( \frac{dp/\,dx}{dp/\,dx}_{\mu_0} \right)^{0.5} \]  

(8)

The Lockhart-Martinelli parameter \( X \) was related to the physical property coefficient \( \Gamma \) for turbulent flow in smooth tubes as

\[ X = \left( \frac{1-x}{x} \right)^{2-n/2} \]  

(9)

Chisholm [17] studied the influence of pipe surface roughness on friction pressure gradient during two-phase flow. He developed an equation that allowed his smooth pipe correlation, Eq. (10), to be extrapolated to rough pipe conditions. His equation was

\[ \frac{B_R}{B_s} = \left[ 0.5 \left(1 + \frac{\mu_s}{\mu_0} \right)^2 + 10^{-600\mu_s/\mu_0} \right]^{0.25-n} \]  

(14)

He made the exponent of the term \( (\mu_s/\mu_0) \) large enough so that this term would be small for the data used in his analysis.
number \( (Fr) \), and the effect of surface tension and the total mass flux by Weber number \( (We) \). His correlation was

\[
\phi_{lo}^2 = E + \frac{3.24FH}{Fr^{0.025}We^{0.035}}
\]  

(15)

\[
E = (1 - x)^{1/2} + x^2 \left( \frac{\rho_lf_{co}}{\rho_g f_{lo}} \right)
\]  

(16)

\[
F = x^{0.79} (1 - x)^{0.224}
\]  

(17)

\[
H = \left( \frac{\rho_l}{\rho_g} \right)^{0.91} \left( \frac{\mu_g}{\mu_l} \right)^{0.19} \left( 1 - \frac{\mu_g}{\mu_l} \right)^{0.7}
\]  

(18)

\[
Fr = \frac{G^2}{gd \rho_m^2}
\]  

(19)

\[
We = \frac{G^2d}{\rho_m \sigma}
\]  

(20)

\[
\rho_m = \left( \frac{x}{\rho_g} + \frac{1 - x}{\rho_l} \right)^{1/3}
\]  

(21)

His correlation was for vertical upward flow and horizontal flow. He made comparisons between the data and the predictions of his correlation. The Friedel correlation had shown very good results in predicting two-phase frictional multiplier \( (\phi_{lo}^2) \) for smooth pipes with \( d > 7 \) mm and does not predict the friction two-phase pressure drops in small diameter pipes accurately, especially for high-reduced pressure. The standard deviation was around 30% for single component flows, and about 40-50% for two-component flows.

Whalley [19] recommended with respect to the previous published correlations that the Friedel correlation should be used for \( \mu/\mu_g < 1 \) 000.

**METHOD**

In the present method, the effect of the mass flux on \( \phi_{lo}^2 \) will be taken into account by introducing the Churchill model to define the Fanning friction factor. This is a modification of the Wallis method [6]. It overcomes the main disadvantage in the Wallis method that \( \phi_{lo}^2 \) is independent on the mass flux (i.e. both small and large mass fluxes give the same results). The Wallis method makes a discrepancy with many investigators who have found that \( \phi_{lo}^2 \) is indeed a function of mass flux, among other things. This method is applied using Maple\textsuperscript{TM} Release 9 software [20]. In addition, comparison with other existing correlations for calculating \( \phi_{lo}^2 \) such as the Wallis correlation based on the homogeneous model without mass effect on \( \phi_{lo}^2 \), the Martinelli-Nelson correlation, the Chisholm correlation, and the Friedel correlation is presented.

Comparison with results from other experimental test facilities for calculating \( \phi_{lo}^2 \) is also presented. Digitizeit software program [21] is used to capture data from old figures.

The two-phase frictional pressure gradient can be related to the Fanning friction factor as follows:

\[
\left( \frac{dp}{dz} \right)_{ff} = 2f_n G^2 \frac{\rho_m}{dp_m}
\]  

(22)

\[
\left( \frac{dp}{dz} \right)_{lo} = 2f_{lo} G^2 \frac{\rho_g}{dp_l}
\]  

(23)

Based on the homogeneous model, Eq. (21) defines the density while the equation of viscosity is

\[
\mu_m = \left( \frac{x}{\mu_g} + \frac{1 - x}{\mu_l} \right)^{-1}
\]  

(24)

Using Eqs. (1), (21), (22), and (23), we obtain

\[
\phi_{lo}^2 = \left( \frac{f_m}{f_{lo}} \right) \left( 1 + x \frac{\rho_l - \rho_g}{\rho_g} \right)
\]  

(25)

To take the effect of the mass flux on \( \phi_{lo}^2 \) into account, the model that was developed by Churchill [22] is introduced to define the Fanning friction factor. The Churchill model was a correlation of the Moody chart [23]. Churchill’s correlation spanned the entire range of laminar, transition, and turbulent flow in pipes. The Churchill model equations that define the Fanning friction factor are

\[
f_m = 2 \left[ \frac{8}{Re_m} \right]^{12} + \frac{1}{(a_m + b_m)^{1/2}}
\]  

(26)

\[
a_m = \left[ \frac{2.457 \ln \left( \frac{1}{7 \ Re_m^{0.6} + (0.27 \epsilon/d)} \right)}{Re_m} \right]^{16}
\]  

(27)

\[
b_m = \left( \frac{37530}{Re_m} \right)^{16}
\]  

(28)

\[
f_{lo} = 2 \left[ \frac{8}{Re_{lo}} \right]^{12} + \frac{1}{(a_{lo} + b_{lo})^{1/2}}
\]  

(29)

\[
a_{lo} = \left[ \frac{2.457 \ln \left( \frac{1}{7 \ Re_{lo}^{0.6} + (0.27 \epsilon/d)} \right)}{Re_{lo}} \right]^{16}
\]  

(30)

\[
b_{lo} = \left( \frac{37530}{Re_{lo}} \right)^{16}
\]  

(31)
The Reynolds number equations are

\[ Re_m = \frac{Gd}{\mu_m} \]  
\[ (32) \]

\[ Re_{lo} = \frac{Gd}{\mu_l} \]  
\[ (33) \]

Substituting from Eqs. (26)-(33) and Eq. (24) into Eq. (25), we can obtain an expression for \( \phi_{lo}^2 \) as a function of mass flux (G), pipe diameter (d), pipe roughness (\( \varepsilon \)), mass quality (x), liquid density (\( \rho_l \)), gas density (\( \rho_g \)), liquid dynamic viscosity (\( \mu_l \)), and gas dynamic viscosity (\( \mu_g \)) respectively.

**RESULTS AND DISCUSSION**

**Effect of Stream Pressure on \( \phi_{lo}^2 \)**

Figure 1 shows effect of stream pressure on \( \phi_{lo}^2 \) with steam-water flow in a smooth pipe at \( G = 1 \times 10^6 \) lb/ft\(^2\).hr (1.356 kg/m\(^2\).s) and \( d = 5 \) mm for the pressure values of 14.7, 100, 500, 1,000, 1,500, 2,000, 2,500, 3,000 and 3,206 psia (1.01, 6.89, 34.4, 68.9, 103, 138, 172, 207 and 221.2 bar) respectively. It can be seen that \( \phi_{lo}^2 \) decreases with increasing stream pressure at a given mass quality. Below the critical pressure (221.2 bar), the present correlation approaches the maximum two-phase frictional multiplier in a smooth consistent manner. At the critical pressure, \( \phi_{lo}^2 \) has a value of 1 for any mass quality value.

**Figure 1 Effect of Stream Pressure on \( \phi_{lo}^2 \)**

**Effect of Mass Flux on \( \phi_{lo}^2 \)**

Figure 2 shows effect of mass flux on \( \phi_{lo}^2 \) with steam-water flow in a smooth pipe at a pressure 100 psia (6.89 bar) and \( d = 5 \) mm for the mass flux values of 0.25x10\(^6\), 1x10\(^6\) and 4x10\(^6\) lb/ft\(^2\).hr (339, 1,356 and 5,424 kg/m\(^2\).s) respectively. It can be seen that \( \phi_{lo}^2 \) increases with increasing the mass flux at a given mass quality. For example, at a mass quality of 80\%, increasing the mass flux from 0.25x10\(^6\) to 1x10\(^6\) lb/ft\(^2\).hr (339 to 1,356 kg/m\(^2\).s) increases \( \phi_{lo}^2 \) from 117 to 126.3. Also, the best improvement in \( \phi_{lo}^2 \) due to the increase of mass flux occurs at a mass quality of 100\%.

**Comparison of Present Correlation with Other Correlations**

Figure 3 shows comparison of present correlation with Wallis correlation, Eq. (5), for steam-water flow in a smooth pipe at a pressure of 100 psia (6.89 bar) and \( d = 5 \) mm for the mass flux values of 0.25x10\(^6\), 1x10\(^6\) and 4x10\(^6\) lb/ft\(^2\).hr (339, 1,356 and 5,424 kg/m\(^2\).s) respectively. It can be seen that the Wallis correlation is very close to the present correlation at \( G = 0.25 \times 10^6 \) lb/ft\(^2\).hr (339 kg/m\(^2\).s) because when the mass flux value becomes low, the effect of mass flux on \( \phi_{lo}^2 \) becomes small and present correlation approaches the Wallis correlation. Also, the Wallis correlation approaches the maximum two-phase frictional multiplier in a smooth consistent manner like the present correlation.

Figure 4 shows comparison of present correlation with Martinelli-Nelson correlation for steam-water flow in a smooth pipe at a pressure of 100 psia (6.89 bar) and \( d = 5 \) mm for the mass flux values of 0.25x10\(^6\), 1x10\(^6\) and 4x10\(^6\) lb/ft\(^2\).hr (339, 1,356 and 5,424 kg/m\(^2\).s) respectively. It can be seen that the Martinelli-Nelson correlation shows a peaking effect at high mass qualities while the present correlation approaches the maximum two-phase frictional multiplier in a smooth consistent manner.
Figure 3 Comparison of Present Correlation with Wallis Correlation at 6.89 bar

Figure 4 Comparison of Present Correlation with Martinelli-Nelson Correlation at 6.89 bar

Figure 5 Comparison of Present Correlation with Chisholm Correlation at 6.89 bar

Figure 6 Comparison of Present Correlation with Friedel Correlation at 6.89 bar

Figure 5 shows comparison of present correlation with Chisholm correlation, Eq. (10), for steam-water flow in a smooth pipe at a pressure of 100 psia (6.89 bar) and \( d = 5 \) mm for the mass flux value of \( 1 \times 10^6 \) (1.356 kg/m².s). The Fanning friction factor is represented by the Blasius equation with \( n = 0.25 \) in the Chisholm correlation. It can be seen that the Chisholm correlation shows a peaking effect at high mass qualities while the present correlation approaches the maximum two-phase frictional multiplier in a smooth consistent manner. At a mass quality of 100%, both the Chisholm correlation and the Wallis Correlation have the same value. The Friedel correlation shows better agreement with the present correlation than both the Martinelli-Nelson correlation and the Chisholm correlation.

Figure 6 shows comparison of present correlation with Friedel correlation, Eqs. (15)-(21), for steam-water flow in a smooth pipe at a pressure of 100 psia (6.89 bar) and \( d = 5 \) mm for the mass flux value of \( 1 \times 10^6 \) (1.356 kg/m².s). The Fanning friction factor is represented by the Blasius equation with \( n = 0.25 \) in the Friedel correlation. It can be seen that the Friedel correlation shows a peaking effect at high mass qualities while the present correlation approaches the maximum two-phase frictional multiplier in a smooth consistent manner. At a mass quality of 100%, both the Friedel correlation and the Wallis Correlation have the same value. The Friedel correlation shows better agreement with the present correlation than both the Martinelli-Nelson correlation and the Chisholm correlation.
Comparison of Present Correlation with Results from Other Experimental Test Facilities

Figure 7 shows comparison of experimental data of Janssen and Kervinen [24] for steam-water flow in a smooth pipe at a pressure of 1 066 psia (73.5 bar) and \( d = 0.742 \) in. for \( G = 1.68 \times 10^6 \) lb/ft\(^2\).hr (2 278 kg/m\(^2\).s) with the Martinelli-Nelson correlation and the present correlation. It can be seen that the Martinelli-Nelson correlation shows substantially higher values of \( \phi_{lo}^2 \) than Janssen-Kervinen data, while the present correlation shows good agreement.

Figure 7 Comparison of Present Correlation with Janssen-Kervinen Data [24] at 73.5 bar

SUMMARY AND CONCLUSIONS

A simple method is developed for calculating \( \phi_{lo}^2 \). The homogeneous model is used to calculate the fluid properties. The Churchill model is used to define the Fanning friction factor to include the effect of the mass flux on \( \phi_{lo}^2 \). Effect of stream pressure on \( \phi_{lo}^2 \) is also investigated. Comparison with other existing correlations for calculating \( \phi_{lo}^2 \) such as the Wallis correlation, the Martinelli-Nelson correlation, the Chisholm correlation, and the Friedel correlation is presented. Comparison with results from other experimental test facilities for calculating \( \phi_{lo}^2 \) is also presented. The following conclusions can be drawn based upon the present study:

1. \( \phi_{lo}^2 \) decreases with increasing the stream pressure at a given mass quality and reaches 1 at the critical pressure.

2. \( \phi_{lo}^2 \) increases with increasing the mass flux at a given mass quality. The best improvement in \( \phi_{lo}^2 \) due to the increase of mass flux occurs at a mass quality of 100%.

3. When the mass flux value becomes low, the effect of mass flux on \( \phi_{lo}^2 \) becomes small and present correlation approaches the Wallis correlation.

4. Both the present correlation and the Wallis correlation approach the maximum two-phase frictional multiplier in a smooth consistent manner while the other correlations show a peaking effect at high mass qualities.

5. The Friedel correlation shows better agreement with the present correlation than both the Martinelli-Nelson correlation and Chisholm correlation.

6. Comparison with other experimental data shows better agreement with present correlation than the Martinelli-Nelson correlation.

ACKNOWLEDGMENTS

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NOMENCLATURE

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Subscripts

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REFERENCES


